

Quiz 1.2: Sample Answers

1. Find

$$\lim_{x \rightarrow 3} 3 + 2x + 7x^2$$

Here, we can just substitute $x = 3$ in to get $3 + 2(3) + 7(3)^2 = 72$.

2. Find

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 5x - 14}$$

A: If we substitute $x=2$, we get $0/0$. So, we first must factor, then cancel:

$$= \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{(x + 7)(x - 2)} = \lim_{x \rightarrow 2} \frac{x + 2}{x + 7}$$

We can now substitute $x = 2$ to get $4/9$.

3. Find

$$\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{3x^3 - 3}$$

If we substitute $x = 1$, we get $0/0$. So, we first must factor, then cancel. Here we use the factoring identity $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$:

$$= \lim_{x \rightarrow 1} \frac{(x + 4)(x + 1)}{3(x - 1)(x^2 + x + 1)} = \lim_{x \rightarrow 1} \frac{x + 4}{3(x^2 + x + 1)}$$

We can now substitute $x = 1$ to get $5/9$.

4. Find

$$\lim_{x \rightarrow -1} \frac{x^4 - 1}{x^2 - 4x - 5}$$

If we substitute $x = -1$, we get $0/0$. So we first must factor, then cancel. In this case, the top expression needs to be factored twice:

$$= \lim_{x \rightarrow -1} \frac{(x^2 - 1)(x^2 + 1)}{(x + 1)(x - 5)} = \lim_{x \rightarrow -1} \frac{(x - 1)(x + 1)(x^2 + 1)}{(x + 1)(x - 5)} = \lim_{x \rightarrow -1} \frac{(x - 1)(x^2 + 1)}{x - 5}$$

We can then substitute $x = -1$ to get $-2/3$.

5. Find

$$\lim_{x \rightarrow 0} \frac{\frac{3}{x+4} - \frac{3}{4}}{x}$$

If we substitute $x = 0$, we get $0/0$. So, we first have to simplify by finding a common denominator, then cancel.

$$= \lim_{x \rightarrow 0} \frac{\frac{12 - (3x+12)}{4(x+4)}}{x} = \lim_{x \rightarrow 0} \frac{-3x}{4(x+4)} * \frac{1}{x} = \lim_{x \rightarrow 0} \frac{-3}{4(x+4)}$$

We can then substitute $x = 0$ to get $-3/16$.

6. Find

$$\lim_{x \rightarrow 1/4} \frac{4x - 4}{2\sqrt{x} - 2}$$

For questions with roots, it is often helpful to multiply top and bottom by the conjugate (in this case, $2\sqrt{x} + 2$). However, for this limit, if we substitute $x = 1/4$ in, we do not get $0/0$, so if we substitute we get the correct value for the limit right away. When we substitute, we get

$$= \frac{4(1/4) - 4}{2\sqrt{(1/4)} - 2} = \frac{1 - 4}{2(1/2) - 2} = \frac{-3}{-1} = 3.$$